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$K^+ \to \pi^+ \nu\overline{\nu}$ AND $K_L^0 \to \mu^+\mu^-$ DECAYS WITHIN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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ABSTRACT

We present a detailed calculation of the contributions of charginos, scalar quarks, and charged Higgs boson to the $K^+ \to \pi^+ \nu\overline{\nu}$ and $K_L^0 \to \mu^+\mu^-$ decays. We include mixings: that of charginos and that of the scalar partners of the left and right handed top quark. We find that the box contribution to the amplitudes is much smaller than the penguin contribution, which can be $\sim 10\%$ of the Standard Model contribution, even for relatively large SUSY masses. The charged Higgs contribution can be as large as 25% of the SM contribution in the first decay and as much as 40% of the SM contribution in the second decay.

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I. INTRODUCTION

Rare decays have always been a good field to search for new physics. Among those, rare Kaon decays have been longtime favorites [1]. As the mass of the the top quark increases, within the SM those rare decays become essentially *top physics*. In this note, we will consider two closely related processes: $K_L^0 \to \mu^+ \mu^-$ and $K^+ \to \pi^+ \nu \bar{\nu}$. The first branching ratio is now measured with a precision of a few percent [2]: $7.4 \pm 0.4 \times 10^{-9}$. The upper bound on the second one (5.2×10^{-9}) is now getting close to the predicted value in the SM: $\sim 6 \times 10^{-10}$ [1,3-7] for large top quark mass. The E787 group at Brookhaven National Laboratory aims to measure this branching ratio in the near future. Once the mass of the top quark is measured with good precision, these rare decays will open a window on physics beyond the SM. One of the favorite such models is the Minimal Supersymmetric Standard Model (MSSM)[8]. The effects of the MSSM to the decays mentioned above were considered in many papers a while ago [9–13]. However none of them included the full mixing of the charginos and the scalar partner of the left and right handed top quark. In this paper we present a more complete calculation of the SUSY contributions to the $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L^0 \to \mu^+ \mu^-$ within the MSSM. We include the mixing of the charginos as well as the mixing of the scalar partner of the left and right handed top quark, which is proportional to the top quark mass. This last mixing was rightfully neglected in the past but now that the top quark mass of getting very large, one should not neglect it. We neglect all masses of the fermions compared to the SUSY masses except the mass of the top quark, which we will take to be the recently released CDF value of 174 GeV [14].

II. SUSY CORRECTIONS

Constraints and mixing

Before we proceed with the calculation of the decay amplitudes, we should discuss briefly some limits on two important parameters of SUSY; namely μ and m_{g_2}. These symmetry-breaking parameters are independant but since they are used to generate masses for particles, we can put constraints on them from current experimental bounds on SUSY masses. We will use here the masses of the charginos $(\tilde{W}_i)$. Following [8] we define a mixing matrix

$$ X = \begin{pmatrix} m_{g_2} & m_W \sqrt{2} \sin \beta \\ m_W \sqrt{2} \cos \beta & \mu \end{pmatrix} \tag{1} $$

where $\tan \beta = v_2 / v_1$ is the ratio of the vacuum expectation values (vev's) of the scalar Higgs bosons in the MSSM.* One then proceeds to diagonalize this matrix through unitary matrices such that $U^* X V^{-1} = M_D$. One obtains two eigenvalues

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* Note that $\tan \beta = v_1 / v_2$ in [8].

for the masses:

$$m^2_{\tilde{W}_{1,2}} = \frac{1}{2}\Big\{ m^2_{g_2} + \mu^2 + 2m^2_W$$
$$\pm \sqrt{(m^2_{g_2} - \mu^2)^2 + 4m^4_W \cos^2 2\beta + 4m^2_W(m^2_{g_2} + \mu^2 + 2m_{g_2}\mu \sin 2\beta)} \Big\} \tag{2}$$

and the unitary matrices take the form $U = O_-$ and $V = O_+$ if $\det X \geq 0$ while $V = \sigma_3 O_+$ if $\det X < 0$ with

$$O_\pm = \begin{pmatrix} \cos \phi_\pm & \sin \phi_\pm \\ -\sin \phi_\pm & \cos \phi_\pm \end{pmatrix} \tag{3}$$

with

$$\tan 2\phi_- = 2\sqrt{2}m_W(\mu \, sin\beta + m_{g_2} \cos \beta)/(m^2_{g_2} - \mu^2 - 2m^2_W \cos 2\beta)$$
$$\tan 2\phi_+ = 2\sqrt{2}m_W(\mu \cos \beta + m_{g_2} \sin \beta)/(m^2_{g_2} - \mu^2 + 2m^2_W \cos 2\beta) \tag{4}$$

In order to keep $M^{11}_D \leftrightarrow M_+ > M_- \leftrightarrow M^{22}_D$ one must impose $0 \leq \phi_\pm \leq \pi/2$. One gets a relationship between m_{g_2} and μ through the previous roots: we require these to be larger than 50 GeV since charginos ($\tilde{W}_i$) have not been observed. We use $m_W = 80.1 \, GeV$. This gives us Fig.1 for different values of $\tan\beta$. The regions above the upper curves and below the lower curves are allowed; *ie* both roots will be larger than 50 GeV.

We also include the mixing in the scalar top quark sector. The mass matrix we are dealing with has the form

$$M^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m^2_{\text{top}} + 0.35 \, D_Z & -m_{\text{top}}(A_{\text{top}} + \mu \cot \beta) \\ -m_{\text{top}}(A_{\text{top}} + \mu \cot \beta) & m^2_{\tilde{t}_R} + m^2_{\text{top}} + 0.15 \, D_Z \end{pmatrix} \tag{5}$$

where $D_Z = m^2_Z \cos 2\beta$ and A_{top} is a parameter that describes the strenght of non-supersymmetric trilinear scalar interactions; we set $A_{\text{top}} = m_S$, where m_S is the soft SUSY breaking mass term. We also take $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ equal to m_S. Note also that $0.35 = T^{\text{top}}_3 - e_{\text{top}} \sin^2 \theta_W$ and $0.15 = e_{\text{top}} \sin^2 \theta_W$. Instead of working directly with the current eigenstates $\tilde{t}_{L,R}$ we work with the mass eigenstates

$$\tilde{t}_1 = cos\Theta \tilde{t}_L + \sin \Theta \tilde{t}_R \qquad \tilde{t}_2 = -\sin \Theta \tilde{t}_L + \cos \Theta \tilde{t}_R \tag{6}$$

The different components are obtained by diagonalisation of the previous matrix:

$$tan2\Theta = \frac{2m_{\text{top}}(A_{\text{top}} + \mu \cot \beta}{m^2_{\tilde{t}_R} - m^2_{\tilde{t}_L} - 0.20 \, D_Z}$$

The two mass eigenstates $m_{\tilde{t}_{1,2}}$ are given by

$$m^2_{\text{top}} + \frac{1}{2}\left(m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} + 0.5 D_Z \pm \sqrt{(m^2_{\tilde{t}_R} - m^2_{\tilde{t}_L} - 0.20 D_Z)^2 + 4m^2_{\text{top}}(A_{\text{top}} + \mu \cot \beta)^2} \right) \tag{7}$$

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In order to recover the proper masses when there is no mixing, one has to select $m_{\tilde{t}_1}^2 \to$ negative root and $m_{\tilde{t}_2}^2 \to$ positive root. Again, we constrain the angle Θ to the first quadrant. For a more complete description of the mass matrix for the scalar top quarks and the couplings of the Z boson to the scalar top quarks within the scalar top quark mass eigenstates we refer the reader to the literature [16,17]

We now proceed to the loop calculations themselves. The decay modes occur via box and penguin diagrams. In the SM it turns out that the contribution of the box diagram is negligible compared to the contribution of the penguin diagram for a large top quark mass. In the SM the amplitude of the decay $K_L^0 \to \mu^+\mu^-$ via the box diagram is suppressed by a relative factor of 4 compared to the decay $K^+ \to \pi^+\nu\overline{\nu}$ [3], whereas for the penguin diagram there is a relative minus sign and the function $D(y_j, x_t)$ (eq.2.15 in [5]) has to be replaced by $C(x_t)$ (eq.2.14 in [5]).

In the MSSM these decays proceed through the box diagrams in Fig.2 and the penguin diagrams in Fig.3. In Fig.2 we also include the so called mass insertion diagram. In [12] the authors only considered the first diagram in Fig. 2 whereas the authors in [9] put their emphasis on the second one. We include both and take the full coupling as given in Fig.22 and Fig.23 of [15]*.

Box Diagrams

To calculate the box diagrams in Fig.2, we have used the rules given in appendix D of Ref. 8. After a lenghty calculation, the result† from the box diagram for the decay $K^+ \to \pi^+\nu\overline{\nu}$ is given by:

$$iM_{BOX} = \frac{\alpha^2}{4\sin\theta_W^4} K_{td} K_{ts}^* (V_{\tilde{t}} - V_{\tilde{u}}) \overline{u}_\nu \gamma_\mu P_L v_{\overline{\nu}} \overline{v}_s \gamma^\mu P_L u_d \tag{8}$$

$$V_{\tilde{u}} = \sum_{i,j=1,2} U_{i1} U_{j1} V_{i1} V_{j1} [\tilde{F}_{\tilde{l}\tilde{u}}^{ij} + 2_M \tilde{F}_{\tilde{l}\tilde{u}}^{ij}]$$

$$V_{\tilde{t}} = \sum_{i,j=1,2} U_{i1} U_{j1} \left\{ \left[V_{i1} V_{j1} c_\Theta^2 + V_{i2} V_{j2} \frac{m_{top}^2}{2 m_W^2 \sin^2\beta} s_\Theta^2 \right] [\tilde{F}_{\tilde{l}\tilde{t}_1}^{ij} + 2_M \tilde{F}_{\tilde{l}\tilde{t}_1}^{ij}] \right.$$

$$\left. + \left[V_{i1} V_{j1} s_\Theta^2 + V_{i2} V_{j2} \frac{m_{top}^2}{2 m_W^2 \sin^2\beta} c_\Theta^2 \right] [F_{\tilde{l}\tilde{t}_2}^{ij} + 2_M \tilde{F}_{\tilde{l}\tilde{t}_2}^{ij}] \right\}$$

$c_\Theta^2 = \cos^2\Theta$, $s_\Theta^2 = \sin^2\Theta$. $\tilde{F}_{\tilde{l}\tilde{u},\tilde{t}_{1,2}}^{ij}$ and $_M\tilde{F}_{\tilde{l}\tilde{u},\tilde{t}_{1,2}}^{ij}$ are given in the appendix A. $m_{i,j} = m_{\tilde{W}_{i,j}}$ are the mass eigenvalues of the charginos, $m_{\tilde{l}}$ the mass of the scalar leptons and $m_{\tilde{u},\tilde{t}_{1,2}}$ the masses of the scalar up quark and the eigenstates of the scalar top quark including the mixing. V_{ij} and U_{ij} are the diagonalizing matrices of the charginos as given in eq. C19 in [8] taken to be real. One can show that the result of the box diagram for the decay $K_L^0 \to \mu^+\mu^-$ can be obtained from these results simply by replacing $U_{i1} U_{j1}$ with $-V_{i1} V_{j1}$ and $m_{\tilde{l}}$ with $m_{\tilde{\nu}}$.

In the case with no mixing of the charginos and scalar top quark and when neglecting the mass insertion term and dropping the m_{top}^2 terms eq.(8) agrees with

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* In Fig.22 b)+d) and Fig.23 b)+d) γ_5 has to be replaced by $-\gamma_5$

† All our results are given for one neutrino family.

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eq.(1) in [12] up to a factor of 2.* Note also that $\tilde{F}_{l\tilde{u}}^{ii} \equiv -g(\tilde{y}_j, \tilde{y}_u)/\tilde{m}_W^2$ in [12]. Whereas the function g in eq.(2) in [12] is described as the function $g_1(x, y)$ in eq.C.2 in [5] we have that the function $_M\tilde{F}_{l\tilde{u}}^{ii}$ is described by the function $g_0(x, y)$ in eq. C.2 in [5].

The box diagram with the charged Higgs boson and the up quarks is proportional to the lepton masses and therefore negligible.

As it turned out, the box contribution was always much smaller than the penguin contribution (except when the penguin was 0, by *numerical accident*) and totally negligible.

Penguin contributions

When considering penguin diagrams involving the strong coupling constant one usually first considers the diagrams with gluinos and scalar down quarks within the loop. After a lengthy but straightforward calculation one can show that the $\overline{v}_s \gamma_\mu P_L u_d$ is identical to 0 after Feynman integration and one is left with terms proportional to $(q_\mu q_\nu - q^2 g_{\mu\nu})\overline{v}_s \gamma^\nu P_L u_d$ and $i\sigma^{\mu\nu} q_\nu \overline{v}_s(m_s P_L + m_d P_R)u_d$, which since $q^2 \ll m_Z^2$ and $m_{s,d} \ll m_Z$ are totally negligible [18]. This result was also obtained in [12]. The same argument goes when neutralinos and scalar down quarks are taken within the loop. The result for the penguin diagrams in Fig.3 with charginos and scalar quarks within the loop is given by:

$$iM_{\text{Peng.}} = + \frac{g_2^3}{(4\pi)^2 \cos\Theta_W} K_{td} K_{ts}^* (-M_{2\tilde{q}} + M_{2\tilde{\chi}} + M_{SE}) \overline{v}_s \gamma_\mu P_L u_d \tag{9}$$

$$M_{2\tilde{q}} = \sum_{i=1,2} \left\{ V_{i1}^2 \left[(T_{3t}c_\Theta^2 - e_t s_W^2)c_\Theta^2 \tilde{T}_i^{\tilde{t}_1\tilde{t}_1} + (T_{3t}s_\Theta^2 - e_t s_W^2)s_\Theta^2 \tilde{T}_i^{\tilde{t}_2\tilde{t}_2} \right. \right.$$

$$+ 2c_\Theta^2 s_\Theta^2 T_{3t}\tilde{T}_i^{\tilde{t}_1\tilde{t}_2} \Big]$$

$$+ \frac{m_{\text{top}}^2}{2m_W^2 \sin^2\beta} V_{i2}^2 \left[(T_{3t}c_\Theta^2 - e_t s_W^2)s_\Theta^2 \tilde{T}_i^{\tilde{t}_1\tilde{t}_1} + (T_{3t}s_\Theta^2 - e_t s_W^2)c_\Theta^2 \tilde{T}_i^{\tilde{t}_2\tilde{t}_2} \right.$$

$$- 2c_\Theta^2 s_\Theta^2 T_{3t}\tilde{T}_i^{\tilde{t}_1\tilde{t}_2} \Big]$$

$$- \frac{m_{\text{top}}}{\sqrt{2}m_W \sin\beta} V_{i1}V_{i2} 2s_\Theta c_\Theta \left[(T_{3t}c_\Theta^2 - e_t s_W^2)\tilde{T}_i^{\tilde{t}_1\tilde{t}_1} - (T_{3t}s_\Theta^2 - e_t s_W^2)\tilde{T}_i^{\tilde{t}_2\tilde{t}_2} \right.$$

$$- (c_\Theta^2 - s_\Theta^2)T_{3t}\tilde{T}_i^{\tilde{t}_1\tilde{t}_2} \Big] \Bigg\}$$

$$M_{2\tilde{\chi}} = \sum_{i,j=1,2} \left\{ V_{i1}V_{j1}[c_\Theta^2 \tilde{G}_1^{ij} + s_\Theta^2 \tilde{G}_2^{ij}] + \frac{m_{\text{top}}^2}{2m_W^2 \sin^2\beta} V_{i2}V_{j2}[s_\Theta^2 \tilde{G}_1^{ij} + c_\Theta^2 \tilde{G}_2^{ij}] \right\}$$

$$M_{SE} = (T_{3d} - e_d s_W^2) \sum_{i=1,2} \left\{ V_{i1}^2[c_\Theta^2 \tilde{S}_{i1} + s_\Theta^2 \tilde{S}_{i2}] + \frac{m_{\text{top}}^2}{2m_W^2 \sin^2\beta} V_{i2}^2 \{s_\Theta^2 \tilde{S}_{i1} + c_\Theta^2 \tilde{S}_{i2}\} \right\}$$

The functions $\tilde{T}_i^{ab}$, $\tilde{G}_a^{ij}$ and $\tilde{S}_{ia}$ are given in the Appendix B. After the summation of all diagrams the divergencies cancel in a nontrivial way. Furthermore in the case

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* Interchanging of $\mu^+ (\overline{\nu})$ with $\mu^- (\nu)$ leads to the mass insertion term diagrams, which give a different function and thus not only a factor of 2.

of no mixing $c_\theta = 1$ and $U_{ij} = \delta_{ij} = V_{ij}$ we have $-M_{2\tilde{q}} + M_{2\tilde{\chi}} + M_{SE} \equiv 0$ after Feynman integration. Hence with no mixing we would have obtained the same result as if we had taken gluinos or neutralinos and scalar down quarks within the loop; that is, proportional to the down and strange quark masses and q^2 and therefore negligible compared to the SM result.

To obtain the amplitude of the decay $K^+ \to \pi^+ \nu \overline{\nu}$ we have to multiply eq.9 by the factor $\dfrac{g_2}{2m_Z^2 \cos \Theta_W}$. The amplitude of the decay $K_L^0 \to \mu^+ \mu^-$ differs by an overall minus sign. These contributions are shown relative to the SM for different combinations of masses in Figs.4 and 5. We do not include the quark mixing matrix elements $K_{td} K_{ts}^*$ and consider only the effects of masses and couplings. We will discuss the effect of mixing matrix elements later. One can see that for $\tan \beta \sim 1$ the contributions can be substantial, even for rather large SUSY *masses*. However, when $\tan \beta \neq 1$ the two scales (μ and m_{g_2}) must be close and m_S must be relatively small in order to have a significant contribution. This behaviour becomes more pronounced as the numerical values increase. Except for particular combinations, one has to conclude that the contribution will be rather small; unless the SUSY mixing-matrix elements differ vastly from the SM mixing-matrix elements.

Higgs contributions

Finally we present the results of the contribution when the charged Higgs boson and the up quarks are taken within the penguin diagrams. With the couplings of the charged Higgs to the Z boson and up quarks given in Fig.2 and Fig.8 in [15] after summation over all diagrams and Feynman integration striking cancellations occur and the finite result is given by:

$$iM = \frac{\alpha^2}{8 \sin^4 \Theta_W} \frac{m_{\text{top}}^2}{m_W^4} \cot^2 \beta K_{td} K_{ts}^* \frac{m_{\text{top}}^2}{m_{\text{top}}^2 - m_{H^+}^2} \Big[1 - \frac{m_{H^+}^2}{m_{\text{top}}^2 - m_{H^+}^2} \ln \frac{m_{\text{top}}^2}{m_{H^+}^2} \Big] \overline{v}_s \gamma_\mu P_L u_d$$

(10)

which basically is the last term of eq.(B4) with $m_i^2 \leftrightarrow m_{\text{top}}^2$ and $m_k^2 \leftrightarrow m_{H^+}^2$. This result agrees with eq.(3.22) in [20] up to a minus sign, which comes from a relative sign of the couplings of their eq.(2.6) compared with those given in Fig.2 in [15]. This contribution, we plot in Fig.6. We see that it can be substantial in the $K^+ \to \pi^+ \nu \bar{\nu}$: it can go up to 25% for small masses. The $K_L^0 \to \mu^+ \mu^-$ amplitude differs from eq.(10) simply by a minus sign. It can be as large as 40% of the short-distance SM contribution.

Discussion

In all our previous results, we have used $m_{top} = 174~GeV$ and $\alpha = 1/137$. We have not included any mixing matrix element in our figures. It is important to remember that $K_{td} K_{ts}^*$ in eq.(8) and eq.(9) have not necessarily the same values as in the SM. This was shown in eq.(33) in [19]: the Kobayashi–Maskawa matrix in the couplings of the charginos to quarks and scalar quarks is multiplied by another matrix V, which can be parametrized as follows:

$$V = \begin{pmatrix} 1 & \varepsilon & \varepsilon^2 \\ -\varepsilon & 1 & \varepsilon \\ -\varepsilon^2 & -\varepsilon & 1 \end{pmatrix}$$

(11)

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so that $K \equiv V \cdot K_{SM}$. Clearly, if $\epsilon \ll 1$ then $K \sim K_{SM}$. However, with $\varepsilon = 0.5$ $K_{td}K_{ts}^*$ is enhanced by a factor of 209 over the SM value. This enhancement falls quickly as ε decreases: to 28 for $\varepsilon = 0.3$ and to -0.8 for $\varepsilon = 0.1$. Note that because of the uncertainties in K_{SM}, this last factor has a large error. We can use that enhancement to try to put limits on ε. For example, in the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, considering that the amplitude has to be squared, we can put an upper limit of approximately 0.5 if we want the SUSY-penguin contribution to be smaller than the current experimental limit of approximately 10 times the SM contribution. Obviously, this is not very constraining but future data might put interesting contraints on ε because the penguin contribution to that process does not vary much with the different SUSY parameters once they have reached *typical* values. Certainly, one would have to know the Higgs contribution beforehand.

Since the $K_L^0 \rightarrow \mu^+ \mu^-$ decay proceeds mainly via the $\gamma\gamma$ channel [21], we can only say that one has to be carefull in trying to extract KM matrix elements from that decay: the charged Higgs contribtion has the same KM elements as the short-distance SM contribution and can reduce the amplitude by as much as 40%. On the other hand, the penguin-chargino contributions remain small except for some small parts of phase space, and the box-chargino contributions are negligible.

IV. CONCLUSIONS

In this paper we presented the results of the calculation of the 1 loop correction to the decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L^0 \rightarrow \mu^+ \mu^-$ within the MSSM. We gave a complete analysis and included the mixing of the charginos as well as the mixing of the scalar partners of the left and right handed top quark. We have shown that the box diagram contributions are negligible within the MSSM while the penguin diagram contributions are typically a few percent of the SM contributions but can be up to 15-20% for some particular combinations of μ, m_{g_2}, and m_S when $tan\beta \sim 1$ A precise measurement of the process $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ could lead to a constraint on the SUSY mixing matrix (the ε parameter) once the charged Higgs contribution is known or a bound on its mass is obtained.

In both processes, the charged Higgs contributions to the amplitude can be quite large compared to the short-distance contribution from the SM: up to 25% for the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and up to 40% for the $K_L^0 \rightarrow \mu^+ \mu^-$. Once the top quark mass is well measured, the first decay will open the door for the observation of the Higgs contribution or constraints on its mass.

VI. APPENDIX A

For the box diagram we have to calculate the following integrals:

$$F_{abij}^{\mu\nu} := \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 - m_a^2)(k^2 - m_b^2)(k^2 - m_i^2)(k^2 - m_j^2)} \qquad (A0)$$

$$F_{abij}^{\mu\nu} =: \frac{+ig^{\mu\nu}}{4(4\pi)^2} \tilde{F}_{ab}^{ij}$$

$m_i^2 \neq m_j^2 \neq m_a^2 \neq m_b^2$

$$\tilde{F}_{ab}^{ij} = -\frac{1}{(m_j^2 - m_i^2)(m_b^2 - m_a^2)} \Big\{ \frac{1}{(m_i^2 - m_a^2)(m_i^2 - m_b^2)} [m_i^4(m_b^2 \ln \frac{m_b^2}{m_i^2} - m_a^2 \ln \frac{m_a^2}{m_i^2})$$

$$- m_i^2 m_a^2 m_b^2 \ln \frac{m_b^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2) \Big\} \qquad (A1)$$

$m_j^2 \neq m_i^2 \neq m_a^2 = m_b^2$

$$\tilde{F}_{aa}^{ij} = -\frac{1}{(m_j^2 - m_i^2)} \Big\{ \frac{1}{(m_i^2 - m_a^2)} [m_a^2 - \frac{m_i^4}{(m_i^2 - m_a^2)} \ln \frac{m_i^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2) \Big\} \quad (A2)$$

$m_i^2 = m_j^2 \neq m_a^2 \neq m_b^2$

$$\tilde{F}_{ab}^{ii} = \tilde{F}_{aa}^{ij}(m_a^2 \leftrightarrow m_i^2, \; m_b^2 \leftrightarrow m_j^2) \qquad (A3)$$

$m_j^2 = m_i^2 \neq m_a^2 = m_b^2$

$$\tilde{F}_{aa}^{ii} = -\frac{(m_i^2 + m_a^2)}{(m_i^2 - m_a^2)^2} \Big\{ 1 - \frac{2m_i^2 m_a^2}{(m_i^4 - m_a^4)} \ln \frac{m_i^2}{m_a^2} \Big\} \qquad (A4)$$

The second integral is given by:

$$_M F_{ab}^{ij} := \int \frac{d^4k}{(2\pi)^4} \frac{m_i m_j}{(k^2 - m_a^2)(k^2 - m_b^2)(k^2 - m_i^2)(k^2 - m_j^2)} \qquad (A5)$$

$$_M F_{ab}^{ij} =: \frac{-ig^{\mu\nu}}{(4\pi)^2} {}_M \tilde{F}_{ab}^{ij}$$

$m_i^2 \neq m_j^2 \neq m_a^2 \neq m_b^2$

$$_M \tilde{F}_{ab}^{ij} = -\frac{m_i m_j}{(m_j^2 - m_i^2)(m_b^2 - m_a^2)} \Big\{ \frac{1}{(m_i^2 - m_a^2)(m_i^2 - m_b^2)} [m_i^2(m_b^2 \ln \frac{m_b^2}{m_i^2} - m_a^2 \ln \frac{m_a^2}{m_i^2})$$

$$- m_a^2 m_b^2 \ln \frac{m_b^2}{m_a^2}] - (m_i^2 \leftrightarrow m_j^2) \Big\} \qquad (A6)$$

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$$m_j^2 \neq m_i^2 \neq m_a^2 = m_b^2$$

$$_M\tilde{F}_{aa}^{ij} = -\frac{m_i m_j}{(m_j^2 - m_i^2)}\Big\{\frac{1}{(m_i^2 - m_a^2)}\Big[1 - \frac{m_i^2}{(m_i^2 - m_a^2)}\ln\frac{m_i^2}{m_a^2}\Big] - (m_i^2 \leftrightarrow m_j^2)\Big\} \quad (A7)$$

$$m_i^2 = m_j^2 \neq m_a^2 \neq m_b^2$$

$$_M\tilde{F}_{ab}^{ii} = {}_M\tilde{F}_{aa}^{ij}(m_a^2 \leftrightarrow m_i^2,\ m_b^2 \leftrightarrow m_j^2,\ m_i m_j \to m_i^2) \quad (A8)$$

$$m_j^2 = m_i^2 \neq m_a^2 = m_b^2$$

$$_M\tilde{F}_{aa}^{ii} = -\frac{m_i^2}{(m_i^2 - m_a^2)^2}\Big\{2 + \frac{(m_i^2 + m_a^2)}{(m_i^2 - m_a^2)}\ln\frac{m_a^2}{m_i^2}\Big\} \quad (A9)$$

VII. APPENDIX B

For the penguin diagram we have the following integrals:

$$\tilde{T}_i^{kl} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \Big\{\frac{1}{\epsilon} - \gamma + \ln 4\pi\mu^2 -$$

$$\ln[m_i^2 - (m_i^2 - m_k^2)\alpha_1 - (m_i^2 - m_l^2)\alpha_2]\Big\} \quad (B1)$$

$$= \frac{1}{2}\Big\{\frac{1}{\epsilon} - \gamma + \ln 4\pi\mu^2 - \ln m_i^2\Big\} + \frac{3}{4} - \frac{1}{2}\frac{m_l^4}{(m_i^2 - m_l^2)(m_k^2 - m_l^2)}\ln\frac{m_l^2}{m_i^2}$$

$$- \frac{1}{2}\frac{m_k^4}{(m_i^2 - m_k^2)(m_l^2 - m_k^2)}\ln\frac{m_k^2}{m_i^2}\Big\}$$

$$m_k^2 = m_l^2 \neq m_i^2$$

$$\tilde{T}_i^{kk} = \frac{1}{2}\Big\{\frac{1}{\epsilon} - \gamma + \ln 4\pi\mu^2 - \ln m_i^2\Big\} + \frac{3}{4} + \frac{1}{2}\frac{m_k^2}{(m_i^2 - m_k^2)}$$

$$- \frac{1}{2}\frac{m_k^2(m_k^2 - 2m_i^2)}{(m_i^2 - m_k^2)^2}\ln\frac{m_k^2}{m_i^2} \quad (B2)$$

$$\tilde{G}_k^{ij} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \Big\{\Big[\frac{1}{\epsilon} - \gamma - 1 + \ln 4\pi\mu^2$$

$$- \ln[m_k^2 - (m_k^2 - m_i^2)\alpha_1 - (m_k^2 - m_j^2)\alpha_2]\Big]O_{ij}'^L +$$

$$\frac{m_i m_j}{[m_k^2 - (m_k^2 - m_i^2)\alpha_1 - (m_k^2 - m_j)\alpha_2]}O_{ij}'^R\Big\} \quad (B3)$$

$$= [\tilde{T}_i^{kl}(m_k^2 \leftrightarrow m_i^2,\ m_l^2 \leftrightarrow m_j^2) - \frac{1}{2}]O_{ij}'^L$$

$$+ m_i m_j\Big[\frac{m_i^2}{(m_i^2 - m_j^2)(m_i^2 - m_k^2)}\ln\frac{m_i^2}{m_k^2} + \frac{m_j^2}{(m_j^2 - m_i^2)(m_j^2 - m_k^2)}\ln\frac{m_j^2}{m_k^2}\Big]O_{ij}'^R$$

$$m_i^2 = m_j^2 \neq m_k^2$$

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$$\tilde{G}_k^{ii} = [\tilde{T}_i^{kk}(m_k^2 \leftrightarrow m_i^2) - \frac{1}{2}]O_{ii}'^L + \frac{m_i^2}{(m_i^2 - m_k^2)}[1 - \frac{m_k^2}{(m_i^2 - m_k^2)}\ln\frac{m_i^2}{m_k^2}]O_{ii}'^R \qquad (B4)$$

$$\tilde{S}_{ik} = \int\limits_0^1 \alpha_1\{\frac{1}{\epsilon} - \gamma + \ln 4\pi\mu^2 - \ln[m_i^2 - (m_i^2 - m_k^2)\alpha_1]\} \qquad (B5)$$

$$= \frac{1}{2}[\frac{1}{\epsilon} - \gamma + \ln 4\pi\mu^2 - \ln m_i^2] + \frac{1}{4} + \frac{1}{2}\frac{m_i^2}{(m_i^2 - m_k^2)} - \frac{1}{2}(1 - \frac{m_i^4}{(m_i^2 - m_k^2)^2})\ln\frac{m_k^2}{m_i^2}$$

$$O_{ij}'^L = \cos^2\Theta_W \delta_{ij} - \frac{1}{2}V_{i2}V_{j2}$$

$$O_{ij}'^R = \cos^2\Theta_W \delta_{ij} - \frac{1}{2}U_{i2}U_{j2}$$

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FIGURE CAPTIONS

Fig.1 The allowed range on μ and m_{g_2} so that the mass of the charginos will come out larger than 50 GeV. We used $m_W = 80.1$ GeV and $\tan\beta = 1$ (solid line), 2 (long dash), 5 (short dash), 10 (dotted line). The regions above the upper lines and below the lower lines are allowed.

Fig.2 The box diagrams with scalar up quarks and charginos within the loop including the mass insertion diagrams.

Fig.3 The penguin diagrams with scalar up quarks and charginos within the loop. The mass insertion diagram with the Z and chargino couplings leads to the same result and therefore has not to be included. The diagrams with the charged Higgs are obtained by replacing the charginos by the top quark and the scalar top quark by the charged Higgs.

Fig.4 The ratio $Amplitude^{SUSY_{penguin}}/Amplitude^{\mathrm{SM}}$ for the decay $K^+ \to \pi^+\nu\overline{\nu}$ as a function of the scalar mass m_S for $\mu = 200$ (solid line), 500 (dashed line) 1000 (dotted line) GeV and for different values of $tan\beta$. **NB** We do not include quark-mixing matrix elements. In **(A)**, we have $m_{g_2} = 200$ GeV while $m_{g_2} = 500$ GeV in **(B)**.

Fig.5 The same as Fig.4 but for the decay $K^0_L \to \mu^+\mu^-$

Fig.6 The ratio $Amplitude^{SUSY_{Higgs}}/Amplitude^{\mathrm{SM}}$ for the decay $K^+ \to \pi^+\nu\bar\nu$ (dashed line) and for the decay $K^0_L \to \mu^+\mu^-$ (solid line) for $tan\beta = 1$. **NB** We do not include quark-mixing matrix elements.